NEW PERSPECTIVES ON THE GREEN'S FUNCTION FOR QUASI-TEM PLANAR STRUCTURES

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The quasi-static spectral Green's function for various planar structures is decomposed into a fundamental part corresponding to a homogeneous medium and the remainder. This is used to help formulate the quasi-TEM analysis of the coplanar strips whose spectral Green's function is singular and to devise efficient numerical techniques to solve for the quasi-TEM parameters of single and multiple coupled microstrip lines and coplanar strips.

Introduction

The Green's function for certain boundary value problems can be decomposed into a fundamental part having the singularity of the Green's function and a well-behaved regular part[1,2]. This concept is introduced to the quasi-TEM Fourier spectral domain Green's function for planar structures as,

$$\tilde{G}(\alpha) = \tilde{U}(\alpha) + \tilde{V}(\alpha) \,. \tag{1}$$

It is seen that $\tilde{U}(\alpha)$, the fundamental part, is related to the Green's function of the corresponding homogeneous problem and then $\tilde{V}(\alpha)$, the remainder, is a well-behaved function which decays much faster (typically exponentially as fast) than the original Green's function. In addition it is seen that $\tilde{U}(\alpha)$ and more importantly terms involving $\tilde{U}(\alpha)$ in numerical steps can be inverted analytically for a choice of many basis functions. This leads to an accurate solution for the characteristics of certain planar structures for which spectral Green's function is singular at origin such as the coplanar strip lines and enables us to devise efficient schemes to evaluate the numerical quadratures required for the computation of the characteristics of various planar structures.

Some of the results including the formulation of the coplanar strip line problem and numerical results for symmetrical, nonsymmetrical, single, and multiple coupled open microstrip and coplanar strip lines are presented in this paper to illustrate the highlights of the decomposition procedure.

Analysis

The spectral domain Green's functions for the microstrip and the coplanar strip line structures as shown in Fig. 1 are given by,

$$\tilde{G}(\alpha) = \frac{1}{\varepsilon_0|\alpha|(1 + \varepsilon_r \coth|\alpha|h)} \tag{2}$$

for the microstrip lines and

$$\tilde{G}(\alpha) = \frac{1 + \varepsilon_r \coth|\alpha|h}{\varepsilon_0|\alpha|\{(1 + \varepsilon_r^2) + 2\,\varepsilon_r \coth|\alpha|h\}} \tag{3}$$

for the coplanar strips.

As seen from Eq. (3), the Green's function for coplanar lines has an extra singularity as $\alpha \to 0$ in addition to the usual source point singularity implied by $\tilde{G}(\alpha) \sim O(1/\alpha)$ as $\alpha \to \infty$ for both structures. This is perhaps the main reason that the study of the quasi-TEM characteristics of coplanar strip lines has been confined to approximate conformal transformation[3], even though they have been analyzed by the spectral domain method for frequency-dependent characteristics[4] where the problem of singularity does not arise. The quasi-TEM characteristics of these structures are evaluated numerically by solving the following integral equation for unknown charges in terms of given potentials

$$\phi(x) = \int_{-\infty}^{\infty} G(x-x')\,\rho(x')\,dx' \tag{4}$$

This leads to a set of linear equations for which the matrix elements are definite integrals[5]. For example, the use of local basis approximation with choice of gate functions (piece-wise constants) for basis function set and collocation for test function set leads to

$$\phi(x_i) = \sum_{j=1}^{N} K_{ij} a_j \,, \quad j = 1,2,\ldots,N \tag{5}$$

$$\text{where } K_{ij} = (1/\pi) \int_0^{\infty} \tilde{G}(\alpha)\tilde{\Pi}(\alpha)\cos(x_i - x_j)\alpha \, d\alpha \tag{6}$$

$$\text{with } \tilde{\Pi}(\alpha) = \frac{2}{b\alpha}\sin\left(\frac{b\alpha}{2}\right) .$$

The local basis approximation method is used here simply because it is readily adapted to the problem of nonsymmetrical and general multiple strips. We now consider the spectral Green's function for the two cases as given by Eqs. (2) and (3).

Microstrip Lines

The Green's function of Eq. (2) is decomposed as,

$$\tilde{G}(\alpha) = \frac{2}{1+\varepsilon_r}\tilde{G}_0(\alpha) + \frac{1-\varepsilon_r}{1+\varepsilon_r} e^{-2|\alpha|h}\,\tilde{G}(\alpha) = \tilde{U}(\alpha) + \tilde{V}(\alpha) \tag{7}$$

where $\tilde{G}_0(\alpha) = \dfrac{1}{\varepsilon_0|\alpha|(1+\coth|\alpha|h)}$,

is the corresponding homogeneous Green's function.

With the above decomposition, it is seen that $\tilde{U}(\alpha)$ and $\tilde{U}(\alpha)\tilde{\Pi}(\alpha)$ can be inverted analytically and $\tilde{V}(\alpha)$ decays exponentially faster than $\tilde{G}(\alpha)$ making the evaluation of the integrals as given by Eq. (6) much faster. With the above decomposition and utilizing the expression for the inverse transform of $\tilde{G}(\alpha)$ as given by

$$F^{-1}[G_0(\alpha)] = \frac{1}{4\pi\varepsilon_0}\log\frac{4h^2+x^2}{x^2} \,, \tag{8}$$

the matrix coefficients K_{ij} are found to be:

$$K_{ij} = \frac{[I_n - I_{n-1}]}{2\pi\varepsilon_0 b(1+\varepsilon_r)} + \frac{1}{\pi}\int_0^{\infty}\tilde{V}(\alpha)\tilde{\Pi}(\alpha)\cos(nb\alpha)d\alpha \tag{9}$$

with

$$I_n \triangleq (n+\tfrac{1}{2})b\,\log\left\{1 + \frac{4h^2}{[(n+\frac{1}{2})b]^2}\right\} + 4h\tan^{-1}\frac{(n+\frac{1}{2})b}{2h}$$

and $n \triangleq i-j$

The latter integral term in Eq. (9) converges quite rapidly. It should be noted that, once the terms involving $\tilde{G}_0(\alpha)$ are evaluated for homogeneous air medium, they need not be recomputed for the inhomogeneous case. Therefore, the decomposition of the inhomogeneous Green's function leads to efficient numerical schemes by replacing the slowly convergent integrals with rapidly converging ones and by utilizing the analytical results as much as possible while avoiding redundant computations.

Another feature of Eq. (7) and similar ones for other simple structures worth mentioning is that the decomposition process is iterative in nature. Thus, $\tilde{G}(\alpha)$ can be expanded in an infinite convergent series, and then, it can be inverted term by term analytically to give the spatial Green's function which is otherwise derived from the image method[7] as follows.

$$\tilde{G}(\alpha) = \frac{1}{1+\varepsilon_r}\sum_{n=0}^{\infty}\left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)^n e^{-2|\alpha|hn}\,\tilde{G}_0(\alpha) \qquad (10)$$

Each term when inverted by utilizing Eq. (8) leads to

$$G(x-x') = \frac{1}{2\pi\varepsilon_0(1+\varepsilon_r)}\sum_{n=0}^{\infty}\left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)^n \log\left\{\frac{[(2n+1)h]^2+(x-x')^2}{(2nh)^2+(x-x')^2}\right\} \qquad (11)$$

This equivalence between the spectral and spatial Green's function has not been explicitly shown to our knowledge beyond the implicit understanding of its inevitability.

Coplanar Strip Lines

Direct integration of Eq. (6) with $\tilde{G}(\alpha)$ as given by Eq. (3) is not feasible because of the singularity at $\alpha = 0$. This singularity in $\tilde{G}(\alpha)$ as $\alpha \to 0$ can however be resolved by the utilization of the regularization of generalized function concept[6]. It is seen that this is physically equivalent to defining the potential with reference to some finite point in space. Here, we decompose the Green's function such that the integrals like Eq. (6) are decomposed into a part with singularity which is evaluated analytically by using the regularization or directly by a simple physical example, and the remainder which is a well-behaved function. The use of decomposition gives

$$\tilde{G}(\alpha) = \frac{1}{2\varepsilon_0|\alpha|} + \frac{(1-\varepsilon_r^2)}{2(1+\varepsilon_r^2)\varepsilon_0|\alpha|\left(1+\frac{2\varepsilon_r}{1+\varepsilon_r^2}\coth|\alpha|h\right)} \qquad (12)$$

$$\triangleq \tilde{U}(\alpha) + \tilde{V}(\alpha)$$

Even though $\tilde{U}(\alpha)$ which is the corresponding homogeneous Green's function is singular, its inverse transform is known by[6]

$$F^{-1}[\tilde{U}(\alpha)] = C - 1/(2\pi\varepsilon_0)\log|x-x'| \; . \qquad (13)$$

In addition, terms involving $\tilde{U}(\alpha)$ like $\tilde{U}(\alpha)\tilde{\Pi}(\alpha)$ or $\tilde{G}(\alpha)\tilde{\Pi}^2$ can be analytically inverted. These are required to solve for the characteristics of coplanar strip lines, e.g., Eq. (5). For example, the matrix elements K_{ij} can be decomposed as

$$K_{ij} = \frac{1}{2\pi\varepsilon_0}\left(E_{n-1} - E_n\right) + \frac{1}{\pi}\int_0^{\infty}\tilde{V}(\alpha)\tilde{\Pi}(\alpha)\cos(nb\alpha)d\alpha \qquad (14)$$

where $n \triangleq i-j$ and $E_n \triangleq (n+1/2)\log|n+1/2|b$.

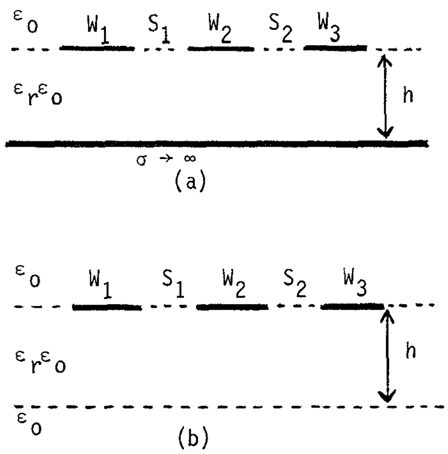

Fig. 1. Schematics. (a) Microstrips; (b) Coplanar Strips.

Note that $V(\alpha)$ is of the same form as Eq. (2) and the later integral term in Eq. (14) can be decomposed further into a term that is inverted analytically and a rapidly converging term.

Results

The numerical results for some typical microstrip and coplanar stip structures are shown in Figs. 2 through 5. The results for symmetrical three-line microstrip structures with equal or unequal strip widths and general single and coupled coplanar strip lines are presented simply to demonstrate the versatility of the computational program which has been implemented on the CDC 6400 computer. For a given geometry and the number of lines, the program computes the quasi-TEM normal mode parameters of the system. These include the phase constants and the characteristic impedances of each line for all the normal modes of the system. The computation time required for each case is only a fraction of a second. Similar results have been obtained by applying the decomposition method to other structures including covered lines and multilayer dielectric medium.

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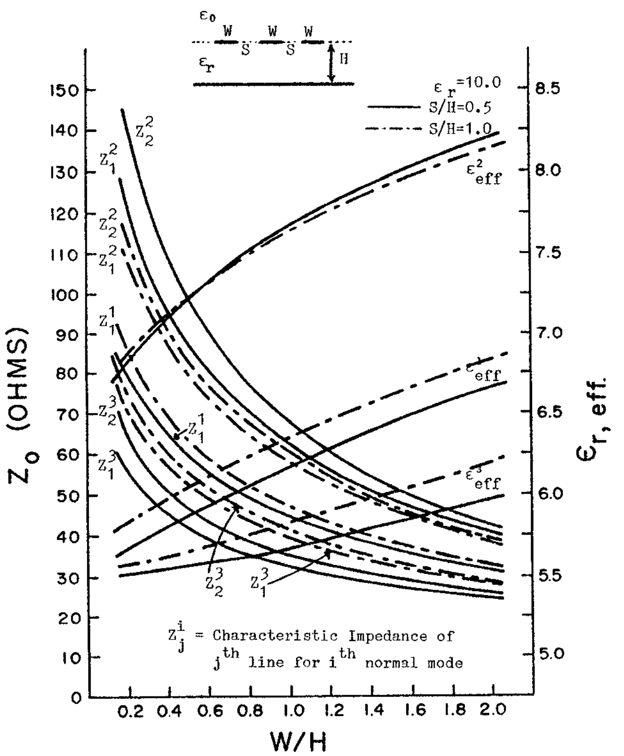

Fig. 2. Microstrip 3-Coupled (Symmetric) Lines (Equal Width)

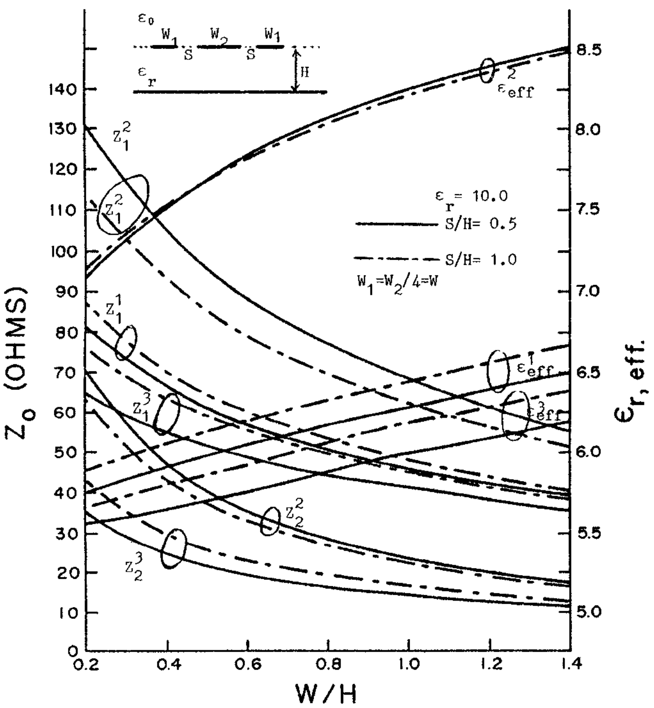

Fig. 3. Microstrip 3-Coupled (Symmetric) Lines (Unequal Width)

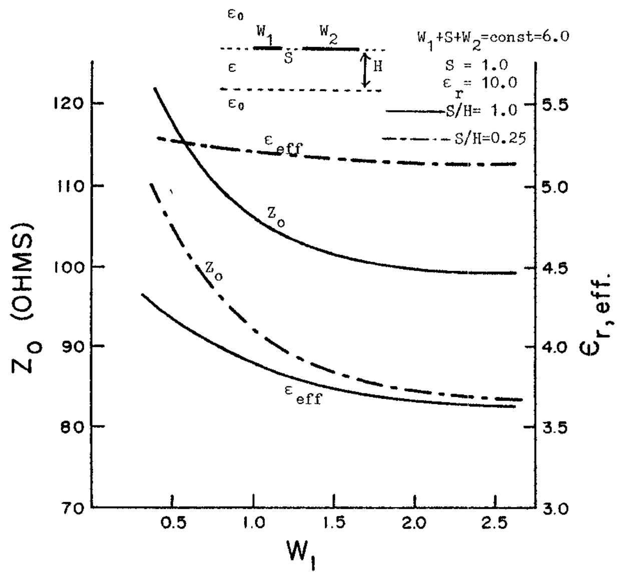

Fig. 4. Coplanar Stripline (Asymmetric Single)

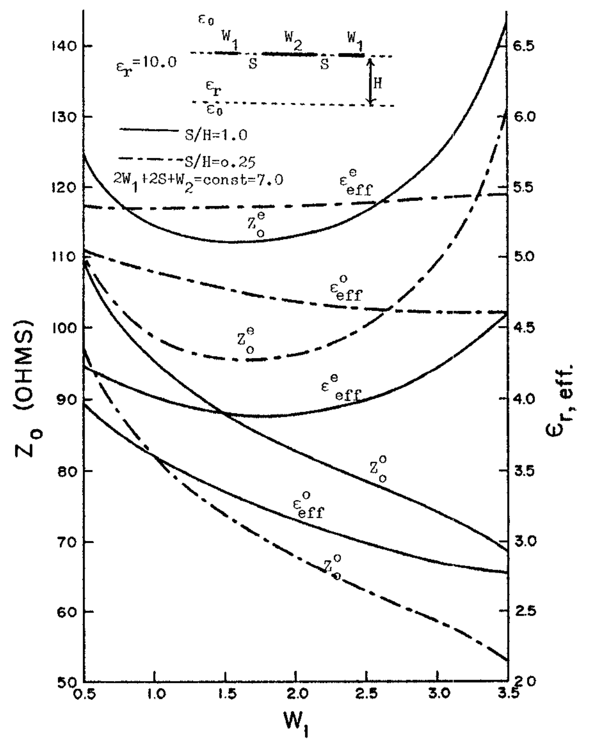

Fig. 5. Coplanar Stripline (Symmetric Coupled Lines) Reference - Line 2